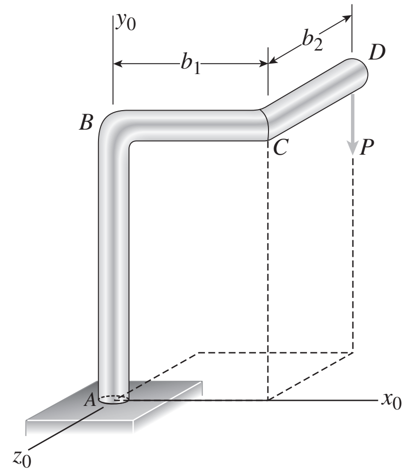
Combined Loadings

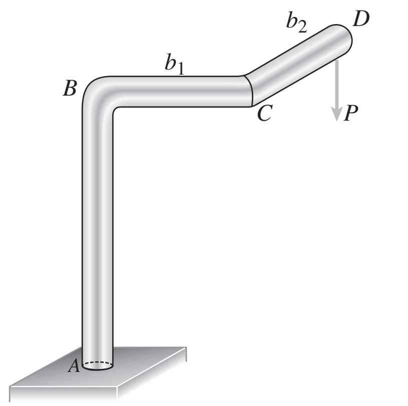
The problems for Section 8.5 are to be solved assuming that the structures behave linearly elastically and that the stresses caused by two or more loads may be superimposed to obtain the resultant stresses acting at a point. Consider both in-plane and out-of-plane shear stresses unless otherwise specified.

Problem 8.5-1 A bracket *ABCD* having a hollow circular cross section consists of a vertical arm *AB*, a horizontal arm *BC* parallel to the x_0 axis, and a horizontal arm *CD* parallel to the z_0 axis (see figure). The arms *BC* and *CD* have lengths $b_1 = 3.2$ ft and $b_2 = 2.4$ ft, respectively. The outer and inner diameters of the bracket are $d_2 = 8.0$ in. and $d_1 =$ 7.0 in. A vertical load $P = 1500$ lb acts at point *D*.

Determine the maximum tensile, compressive, and shear stresses in the vertical arm.

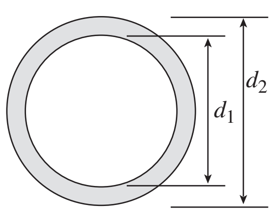


Solution 8.5-1 Bracket ABCD



$b_1 = 3.2$ ft $\quad b_2 = 2.4$ ft $\quad P = 1500$ lb

CROSS SECTION

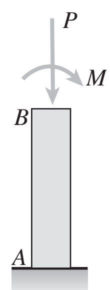


$d_2 = 8.0$ in.
$d_1 = 7.0$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 11.781 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 83.203 \text{ in.}^4$$

VERTICAL ARM AB



$$P = 1500 \text{ lb}$$
$$M = P(\text{distance } B_D)$$
$$= P\sqrt{b_1^2 + b_2^2} = (1500 \text{ lb})(4.0 \text{ ft})$$
$$= 6{,}000 \text{ lb-ft} = 72{,}000 \text{ lb-in.}$$

MAXIMUM STRESSES occur on opposite sides of the vertical arm.

MAXIMUM TENSILE STRESS

$$\sigma_t = -\frac{P}{A} + \frac{M(d_2/2)}{I}$$
$$= -\frac{1500 \text{ lb}}{11.781 \text{ in.}^2} + \frac{(72{,}000 \text{ lb-in.})(4.0 \text{ in.})}{83.203 \text{ in.}^4}$$
$$= -127.3 \text{ psi} + 3461.4 \text{ psi} = 3330 \text{ psi} \quad \longleftarrow$$

MAXIMUM COMPRESSIVE STRESS

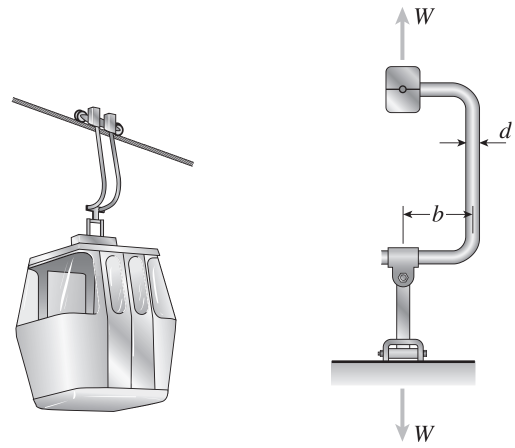
$$\sigma_c = -\frac{P}{A} - \frac{M(d_2/2)}{I} = -127.3 \text{ psi} - 3461.4 \text{ psi}$$
$$= -3590 \text{ psi} \quad \longleftarrow$$

MAXIMUM SHEAR STRESS

Uniaxial stress. $\quad \tau_{max} = \left|\frac{\sigma_c}{2}\right| = 1790 \text{ psi} \quad \longleftarrow$

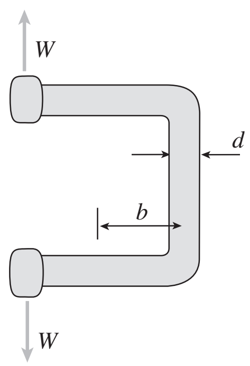
Problem 8.5-2 A gondola on a ski lift is supported by two bent arms, as shown in the figure. Each arm is offset by the distance $b = 180$ mm from the line of action of the weight force W. The allowable stresses in the arms are 100 MPa in tension and 50 MPa in shear.

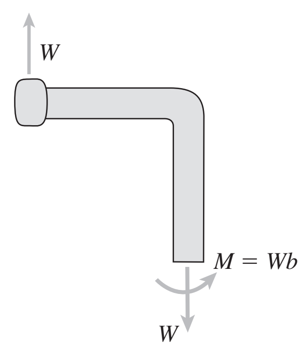
If the loaded gondola weighs 12 kN, what is the mininum diameter d of the arms?



(a) (b)

Solution 8.5-2 **Gondola on a ski lift**





$b = 180 \text{ mm} \qquad W = \dfrac{12 \text{ kN}}{2} = 6 \text{ kN}$

$\sigma_{allow} = 100 \text{ MPa (tension)} \qquad \tau_{allow} = 50 \text{ MPa}$

Find d_{min}

$A = \dfrac{\pi d^2}{4} \qquad S = \dfrac{\pi d^3}{32}$

MAXIMUM TENSILE STRESS

$$\sigma_t = \frac{W}{A} + \frac{M}{S} = \frac{4W}{\pi d^2} + \frac{32\, Wb}{\pi d^3}$$

$$\text{or} \left(\frac{\pi \sigma_t}{4W}\right) d^3 - d - 8b = 0$$

SUBSTITUTE NUMERICAL VALUES:

$$\frac{\pi \sigma_t}{4W} = \frac{\pi \sigma_{allow}}{4W} = \frac{\pi(100 \text{ MPa})}{4(6 \text{ kN})} = 13{,}089.97 \frac{1}{m^2}$$

$8b = 1.44$ m

$13{,}090\, d^3 - d - 1.44 = 0 \qquad (d = \text{meters})$

Solve numerically: $d = 0.04845$ m

$\therefore\ d_{min} = 48.4$ mm ←

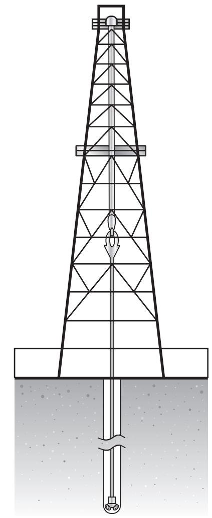
MAXIMUM SHEAR STRESS

$$\tau_{max} = \frac{\sigma_t}{2} \text{ (uniaxial stress)}$$

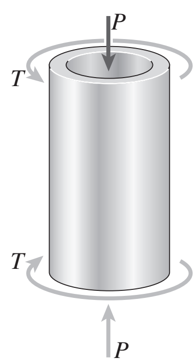
Since τ_{allow} is one-half of σ_{allow}, the minimum diameter for shear is the same as for tension.

Problem 8.5-3 The hollow drill pipe for an oil well (see figure) is 6.0 in. in outer diameter and 0.75 in. in thickness. Just above the bit, the compressive force in the pipe (due to the weight of the pipe) is 60 k and the torque (due to drilling) is 170 k-in.

Determine the maximum tensile, compressive, and shear stresses in the drill pipe.



Solution 8.5-3 Drill pipe for an oil wall



P = compressive force

T = torque $P = 60$ k $T = 170$ k-in.

d_2 = outer diameter t = thickness

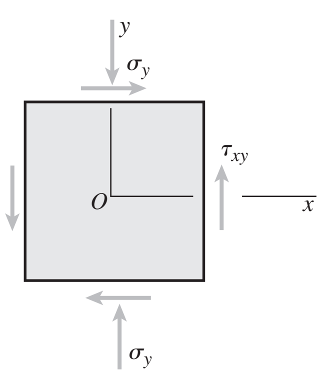
d_1 = inner diameter $d_2 = 6.0$ in. $t = 0.75$ in.

$d_1 = d_2 - 2t = 4.5$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 12.370 \text{ in.}^2$$

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 86.977 \text{ in.}^4$$

STRESSES AT THE OUTER SURFACE



$$\sigma_y = -\frac{P}{A} = -\frac{60 \text{ k}}{12.370 \text{ in.}^2} = -4850 \text{ psi}$$

$$\tau_{xy} = \frac{T(d_2/2)}{I_P} = 5864 \text{ psi}$$

$$\sigma_x = 0$$

PRINCIPAL STRESSES

$$\sigma_{1,2} = \frac{\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -2425 \text{ psi} \pm \sqrt{(-2425)^2 + (5864)^2}$$

$$= -2425 \text{ psi} \pm 6345 \text{ psi}$$

$\sigma_1 = 3920$ psi $\sigma_2 = -8770$ psi

MAXIMUM TENSILE STRESS $\sigma_t = 3920$ psi ←

MAXIMUM COMPRESSIVE STRESS

$\sigma_c = -8770$ psi ←

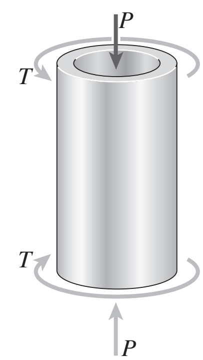
MAXIMUM IN-PLANE SHEAR STRESS

$$\tau_{max} = \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 6350 \text{ psi}$$ ←

NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

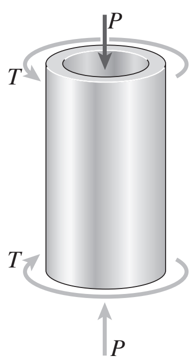
Problem 8.5-4 A segment of a generator shaft is subjected to a torque T and an axial force P, as shown in the figure. The shaft is hollow (outer diameter d_2 = 280 mm and inner diameter d_1 = 230 mm) and delivers 1800 kW at 4.0 Hz.

If the compressive force P = 525 kN, what are the maximum tensile, compressive, and shear stresses in the shaft?



Probs. 8.5-4 and 8.5-5

Solution 8.5-4 **Generator shaft**



(Determine the stresses)

$P = 525$ kN

$d_2 = 280$ mm $\quad d_1 = 230$ mm

P_0 = power $\quad P_0 = 1800$ kw

f = frequency $\quad f = 4.0$ Hz

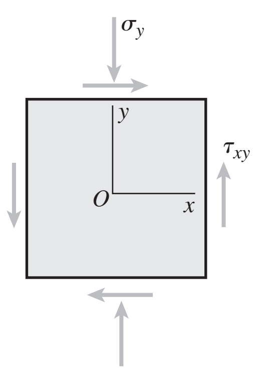
T = torque $\quad T = \dfrac{P_0}{2\pi f}$ (Eq. 3-40)

$$T = \frac{1800 \times 10^3 W}{2\pi(4.0 \text{ Hz})} = 71{,}620 \text{ N} \cdot \text{m}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 20.028 \times 10^{-3} \text{ m}^2$$

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 328.70 \times 10^{-6} \text{ m}^4$$

STRESSES AT THE SURFACE OF THE SHAFT



$$\sigma_y = -\frac{P}{A} = -\frac{525 \text{ kN}}{20.028 \times 10^{-3} \text{ m}^2} = -26.21 \text{ MPa}$$

$$\tau_{xy} = \frac{T(d_2/2)}{I_P} = \frac{(71{,}620 \text{ N} \cdot \text{m})(140 \text{ mm})}{328.70 \times 10^{-6} \text{m}^4}$$

$$= 30.50 \text{ MPa}$$

$$\sigma_x = 0$$

PRINCIPAL STRESSES

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -13.11 \text{ MPa} \pm 33.20 \text{ MPa}$$

$\sigma_1 = 20.1$ MPa $\quad \sigma_2 = -46.3$ MPa

MAXIMUM TENSILE STRESS

$\sigma_t = \sigma_1 = 20.1$ MPa ←

MAXIMUM COMPRESSIVE STRESS

$\sigma_c = \sigma_2 = 46.3$ MPa ←

MAXIMUM IN-PLANE SHEAR STRESS

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

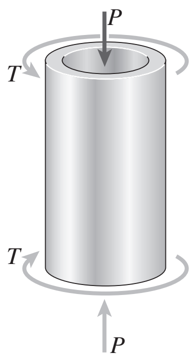
$= 33.2$ MPa ←

NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

Problem 8.5-5 A segment of a generator shaft of hollow circular cross section is subjected to a torque $T = 220$ k-in. (see figure). The outer and inner diameters of the shaft are 8.0 in. and 6.0 in., respectively.

What is the maximum permissible compressive load P that can be applied to the shaft if the allowable in-plane shear stress is $\tau_{allow} = 6500$ psi?

Solution 8.5-5 Generator shaft



(Determine the maximum allowable load P)

$T = 220$ k-in.

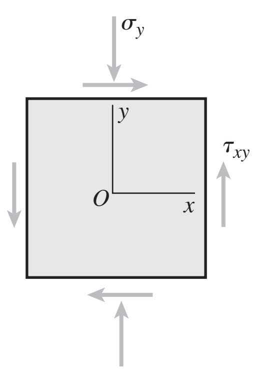
$d_2 = 8.0$ in. $d_1 = 6.0$ in.

$\tau_{allow} = 6500$ psi (In-plane shear stress)

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 21.9911 \text{ in.}^2$$

$$I_P = \frac{\pi}{32}(d_2^4 - d_1^4) = 274.889 \text{ in.}^4$$

STRESSES AT THE SURFACE OF THE SHAFT



$\sigma_x = 0$

$$\sigma_y = -\frac{P}{A} = -\frac{P}{21.9911 \text{ in.}^2}$$

Units: P = pounds

σ_y = psi

$$\tau_{xy} = \frac{T(d_2/2)}{I_P} = \frac{(220 \text{ k-in.})(4.0 \text{ in.})}{274.889 \text{ in.}^4}$$

$= 3,201.29$ psi

MAXIMUM IN-PLANE SHEAR STRESS

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{or} \quad \tau_{allow}^2 = \left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$(6500)^2 = \left[\frac{P}{(2)(21.9911)}\right]^2 + (3,201.29)^2$$

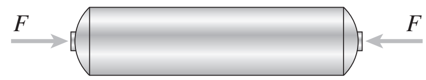
$$\frac{P^2}{1934.43} = 32,001,700; \quad P^2 = 61.905 \times 10^9$$

$P = 248,810$ lb $\quad P_{max} = 249$ k ⟵

NOTE: The maximum in-plane shear stress is larger than the maximum out-of-plane shear stress (in this example).

Problem 8.5-6 A cylindrical tank subjected to internal pressure p is simultaneously compressed by an axial force $F = 72$ kN (see figure). The cylinder has diameter $d = 100$ mm and wall thickness $t = 4$ mm.

Calculate the maximum allowable internal pressure p_{max} based upon an allowable shear stress in the wall of the tank of 60 MPa.



Solution 8.5-6 Cylindrical tank with compressive force

$F = 72$ kN
p = internal pressure
$d = 100$ mm $\quad t = 4$ mm $\quad \tau_{allow} = 60$ MPa

CIRCUMFERENTIAL STRESS (TENSION)

$$\sigma_1 = \frac{pr}{t} = \frac{p(50 \text{ mm})}{4 \text{ mm}} = 12.5\,p$$

Units: σ_1 = MPa $\quad p$ = MPa

LONGITUDINAL STRESS (TENSION)

$$\sigma_2 = \frac{pr}{2t} - \frac{F}{A} = \frac{pr}{2t} - \frac{F}{2\pi rt}$$
$$= 6.25p - \frac{72{,}000\ N}{2\pi(50 \text{ mm})(4 \text{ mm})}$$
$$= 6.25p - 57.296 \text{ MPa}$$

Units: σ_2 = MPa $\quad p$ = MPa

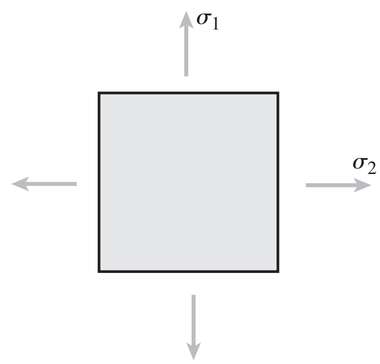
BIAXIAL STRESS

IN-PLANE SHEAR STRESS (CASE 1)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 3.125\,p + 28.648 \text{ MPa}$$

60 MPa = $3.125\,p + 28.648$ MPa

Solving, $p_1 = 10.03$ MPa



OUT-OF-PLANE SHEAR STRESSES

Case 2: $\tau_{max} = \dfrac{\sigma_1}{2} = 6.25\,p$; 60 MPa $= 6.25p$

Solving, $p_2 = 9.60$ MPa

Case 3: $\tau_{max} = \dfrac{\sigma_2}{2} = 3.125\,p - 28.648$ MPa

60 MPa = $3.125\,p - 28.648$ MPa

Solving, $p_3 = 28.37$ MPa

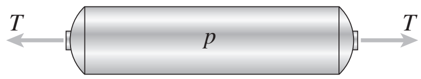
CASE 2, OUT-OF-PLANE SHEAR STRESS GOVERNS

$p_{max} = 9.60$ MPa ←

Problem 8.5-7 A cylindrical tank having diameter $d = 2.5$ in. is subjected to internal gas pressure $p = 600$ psi and an external tensile load $T = 1000$ lb (see figure).

Determine the minimum thickness t of the wall of the tank based upon an allowable shear stress of 3000 psi.

Solution 8.5-7 Cylindrical tank with tensile load



$T = 1000$ lb
$p = 600$ psi
$d = 2.5$ in. $\quad \tau_{allow} = 3000$ psi

CIRCUMFERENTIAL STRESS (TENSION)

$$\sigma_1 = \frac{pr}{t} = \frac{(600\text{ psi})(1.25\text{ in})}{t} = \frac{750}{t}$$

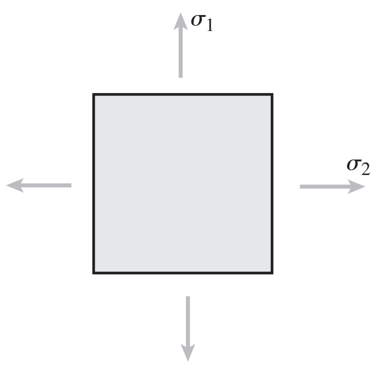
Units: σ_1 = psi $\quad t$ = inches $\quad \sigma_2$ = psi

LONGITUDINAL STRESS (TENSION)

$$\sigma_2 = \frac{pr}{2t} + \frac{T}{A} = \frac{pr}{2t} + \frac{T}{2\pi rt}$$

$$= \frac{375}{t} + \frac{1000\text{ lb}}{2\pi(1.25\text{ in.})t} = \frac{375}{t} + \frac{127.32}{t} = \frac{502.32}{t}$$

BIAXIAL STRESS



IN-PLANE SHEAR STRESS (CASE 1)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{247.68}{2t} = \frac{123.84}{t}$$

$$3000\text{ psi} = \frac{123.84}{t}$$

Solving, $t_1 = 0.0413$ in.

OUT-OF-PLANE SHEAR STRESSES

Case 2: $\tau_{max} = \frac{\sigma_1}{2} = \frac{375}{t}$; $3000 = \frac{375}{t}$

Solving, $t_2 = 0.125$ in.

Case 3: $\tau_{max} = \frac{\sigma_2}{2} = \frac{251.16}{t}$; $3000 = \frac{251.16}{t}$

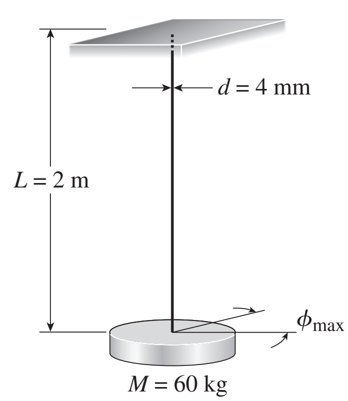
Solving, $t_3 = 0.0837$ in.

CASE 2, OUT-OF-PLANE SHEAR STRESS GOVERNS

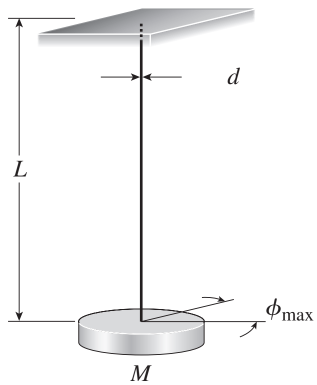
$t_{min} = 0.125$ in. ←

Problem 8.5-8 The torsional pendulum shown in the figure consists of a horizontal circular disk of mass $M = 60$ kg suspended by a vertical steel wire ($G = 80$ GPa) of length $L = 2$ m and diameter $d = 4$ mm.

Calculate the maximum permissible angle of rotation ϕ_{max} of the disk (that is, the maximum amplitude of torsional vibrations) so that the stresses in the wire do not exceed 100 MPa in tension or 50 MPa in shear.



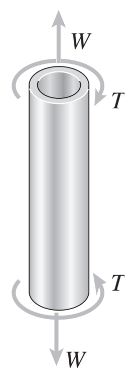
Solution 8.5-8 Torsional pendulum



$L = 2.0$ m $\quad d = 4.0$ mm
$M = 60$ kg $\quad G = 80$ GPa
$\sigma_{allow} = 100$ MPa $\quad \tau_{allow} = 50$ MPa

$$A = \frac{\pi d^2}{4} = 12.5664 \text{ mm}^2$$

$$W = Mg = (60 \text{ kg})(9.81 \text{ m/s}^2) = 588.6\ N$$



TORQUE: $T = \dfrac{GI_p\,\phi_{max}}{L}$ (EQ. 3-15)

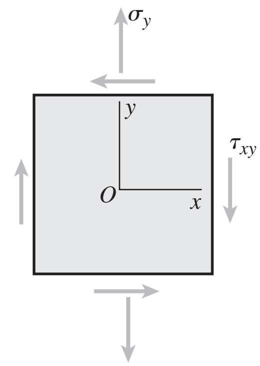
SHEAR STRESS: $\tau = \dfrac{T_r}{I_p}$ (EQ. 3-11)

$$\tau = \left(\frac{GI_p\,\phi_{max}}{L}\right)\left(\frac{r}{I_P}\right) = \frac{Gr\,\phi_{max}}{L} = (80 \times 10^6\ Pa)\phi_{max}$$

$\tau = 80\,\phi_{max}$ $\quad$ Units: τ = MPa $\quad \phi_{max}$ = radius

TENSILE STRESS $\quad \sigma_x = \dfrac{W}{A} = 46.839$ MPa

$\sigma_x = 0$
$\sigma_y = \sigma_t = 46.839$ MPa
$\tau_{xy} = -80\,\phi_{max}$ (MPa)



PRINCIPAL STRESSES

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = 23.420 \pm \sqrt{(23.420)^2 + 6400\,\phi_{max}^2} \quad \text{(MPa)}$$

Note that σ_1 is positive and σ_2 is negative. Therefore, the maximum in-plane shear stress is greater than the maximum out-of-plane shear stress.

MAXIMUM ANGLE OF ROTATION BASED ON TENSILE STRESS

σ_1 = maximum tensile stress $\quad \sigma_{allow} = 100$ MPa

$$\therefore 100 \text{ MPa} = 23.420 + \sqrt{(23.420)^2 + 6400\,\phi_{max}^2}$$

$$(100 - 23.420)^2 = (23.420)^2 + 6400\,\phi_{max}^2$$

$$5316 = 6400\,\phi_{max}^2 \qquad \phi_{max} = 0.9114 \text{ rad} = 52.2^\circ$$

MAXIMUM ANGLE OF ROTATION BASED ON IN-PLANE SHEAR STRESS

$$\tau_{max}\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(23.420)^2 + 6400\,\phi_{max}^2}$$

$\tau_{allow} = 50$ MPa $\quad 50 = \sqrt{(23.420)^2 + 6400\,\phi_{max}^2}$

$$(50)^2 = (23.420)^2 + 6400\,\phi_{max}^2$$

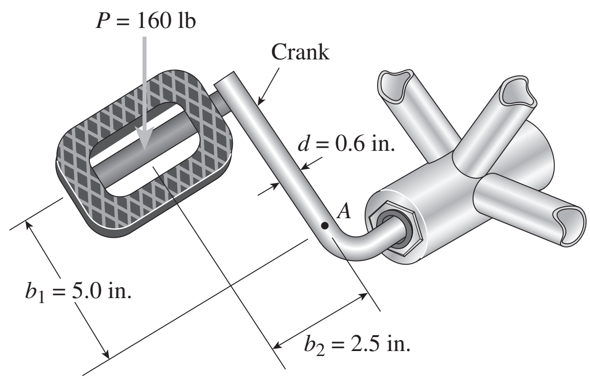
Solving, $\phi_{max} = 0.5522$ rad $= 31.6^\circ$

SHEAR STRESS GOVERNS

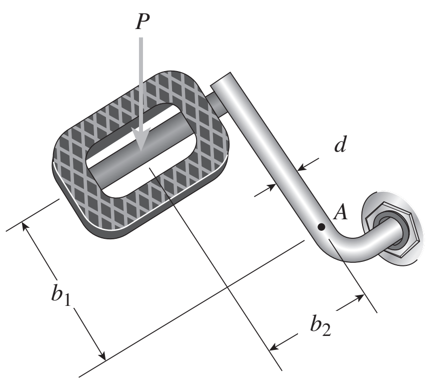
$\phi_{max} = 0.552$ rad $= 31.6^\circ$ ⟵

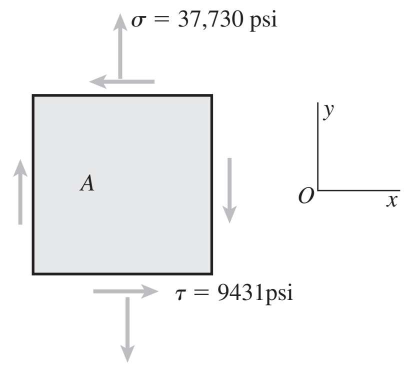
Problem 8.5-9 Determine the maximum tensile, compressive, and shear stresses at point A on the bicycle pedal crank shown in the figure.

The pedal and crank are in a horizontal plane and point A is located on the top of the crank. The load $P = 160$ lb acts in the vertical direction and the distances (in the horizontal plane) between the line of action of the load and point A are $b_1 = 5.0$ in. and $b_2 = 2.5$ in. Assume that the crank has a solid circular cross section with diameter $d = 0.6$ in.



Solution 8.5-9 **Pedal crank**





$P = 160$ lb $\quad d = 0.6$ in.
$b_1 = 5.0$ in. $\quad b_2 = 2.5$ in.

STRESS RESULTANTS on cross section at point A:

Torque: $T = Pb_2 = 400$ lb-in.
Moment: $M = Pb_1 = 800$ lb-in.
Shear force: $V = P = 160$ lb

STRESSES AT POINT A

$$\tau = \frac{16T}{\pi d^3} = 9431 \text{ psi} \qquad \sigma = \frac{M}{S} = \frac{32M}{\pi d^3} = 37{,}730 \text{ psi}$$

(The shear force V produces no shear stresses at point A.)

PRINCIPAL STRESSES AND MAXIMUM SHEAR STRESS

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_x = 0 \quad \sigma_y = 37{,}730$ psi $\quad \tau_{xy} = -9431$ psi

$\sigma_{1,2} = 18{,}860$ psi $\pm$ 21,090 psi
$\sigma_1 = 39{,}950$ psi $\quad \sigma_2 = -2230$ psi

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21{,}090 \text{ psi}$$

MAXIMUM TENSILE STRESS: $\sigma_t = 39{,}950$ psi ⟵

MAXIMUM COMPRESSIVE STRESS:

$\sigma_c = -2{,}230$ psi ⟵

MAXIMUM IN-PLANE SHEAR STRESS:

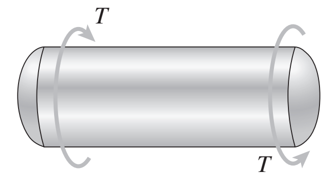
$\tau_{max} = 21{,}090$ psi ⟵

NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

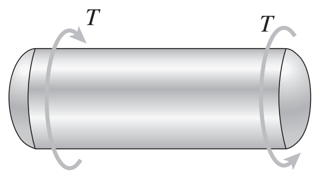
Problem 8.5-10 A cylindrical pressure vessel having radius $r = 300$ mm and wall thickness $t = 15$ mm is subjected to internal pressure $p = 2.5$ MPa. In addition, a torque $T = 120$ kN·m acts at each end of the cylinder (see figure).

(a) Determine the maximum tensile stress σ_{max} and the maximum in-plane shear stress τ_{max} in the wall of the cylinder.

(b) If the allowable in-plane shear stress is 30 MPa, what is the maximum allowable torque T?



Solution 8.5-10 **Cylindrical pressure vessel**



$T = 120 \text{ kN} \cdot \text{m} \qquad r = 300 \text{ mm}$
$t = 15 \text{ mm} \qquad P = 2.5 \text{ MPa}$

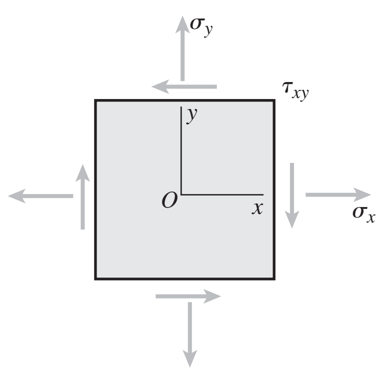
STRESSES IN THE WALL OF THE VESSEL

$$\sigma_x = \frac{pr}{2t} = 25 \text{ MPa} \qquad \sigma_y = \frac{pr}{t} = 50 \text{ MPa}$$

$$\tau_{xy} = -\frac{Tr}{Ip} \quad \text{(Eq. 3-11)} \qquad I_p = 2\pi r^3 t \quad \text{(Eq. 3-18)}$$

$$\tau_{xy} = -\frac{T}{2\pi r^2 t} = -14.147 \text{ MPa}$$

(a) PRINCIPAL STRESSES



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 37.5 \pm 18.878 \text{ MPa}$$

$$\sigma_1 = 56.4 \text{ MPa} \qquad \sigma_2 = 18.6 \text{ MPa}$$

$$\therefore \sigma_{max} = 56.4 \text{ MPa} \quad \longleftarrow$$

Maximum IN-PLANE SHEAR STRESS

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 18.9 \text{ MPa} \quad \longleftarrow$$

(b) MAXIMUM ALLOWABLE TORQUE T

$\tau_{allow} = 30$ MPa (in-plane shear stress)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (1)$$

$$\sigma_x = \frac{pr}{2t} = 25 \text{ MPa} \qquad \sigma_y = \frac{pr}{t} = 50 \text{ MPa}$$

$$\tau_{xy} = -\frac{T}{2\pi r^2 t} = -117.893 \times 10^{-6}\, T$$

Units: $\tau_{xy} = \text{MPa} \qquad T = \text{N} \cdot \text{m}$

Substitute into Eq. (1):

$$\tau_{max} = \tau_{allow} = 30 \text{ MPa}$$

$$= \sqrt{(-12.5 \text{ MPa})^2 + (-117.893 \times 10^{-6}\, T)^2}$$

Square both sides, rearrange, and solve for T:

$$(30)^2 = (12.5)^2 + (117.893 \times 10^{-6})^2\, T^2$$

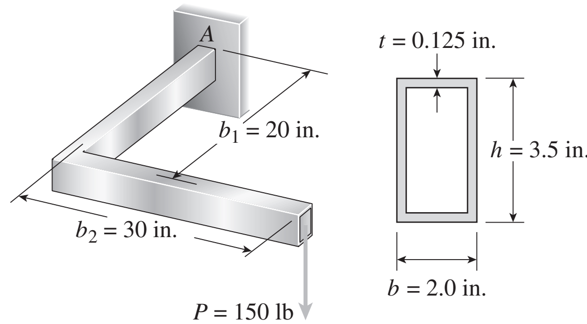
$$T^2 = \frac{743.750}{13{,}899 \times 10^{-12}} = 53{,}512 \times 10^6 \ (\text{N} \cdot \text{m})^2$$

$$T = 231.3 \times 10^3 \text{ N} \cdot \text{m}$$

$$T_{max} = 231 \text{ kN} \cdot \text{m} \quad \longleftarrow$$

Problem 8.5-11 An L-shaped bracket lying in a horizontal plane supports a load $P = 150$ lb (see figure). The bracket has a hollow rectangular cross section with thickness $t = 0.125$ in. and outer dimensions $b = 2.0$ in. and $h = 3.5$ in. The centerline lengths of the arms are $b_1 = 20$ in. and $b_2 = 30$ in.

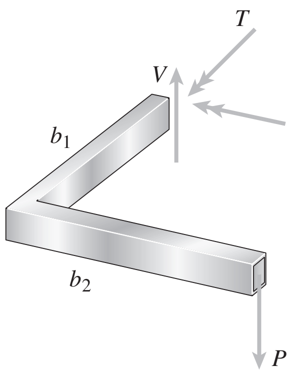
Considering only the load P, calculate the maximum tensile stress σ_t, maximum compressive stress σ_c, and maximum shear stress τ_{max} at point A, which is located on the top of the bracket at the support.



Solution 8.5-11 **L-shaped bracket**

$P = 150$ lb $b_1 = 20$ in. $b_2 = 30$ in.
$t = 0.125$ in. $h = 3.5$ in. $b = 2.0$ in.

FREE-BODY DIAGRAM OF BRACKET



STRESS RESULTANTS AT THE SUPPORT

Torque: $T = Pb_2 = (150 \text{ lb})(30 \text{ in.}) = 4500$ lb-in.
Moment: $M = Pb_1 = (150 \text{ lb})(20 \text{ in.}) = 3000$ lb-in.
Shear force: $V = P = 150$ lb

PROPERTIES OF THE CROSS SECTION

For torsion:

$$A_m = (b - t)(h - t) = (1.875 \text{ in.})(3.375 \text{ in.}) = 6.3281 \text{ in.}^2$$

For bending: $c = \dfrac{h}{2} = 1.75$ in.

$$I = \frac{1}{12}(bh^3) - \frac{1}{12}(b - 2t)(h - 2t)^3$$
$$= \frac{1}{12}(2.0 \text{ in.})(3.5 \text{ in.})^3 - \frac{1}{12}(1.75 \text{ in.})(3.25 \text{ in.})^3$$
$$= 2.1396 \text{ in.}^4$$

STRESSES AT POINT A ON THE TOP OF THE BRACKET

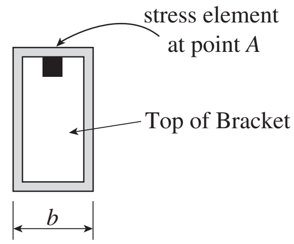
$$\tau = \frac{T}{2tA_m} = \frac{4500 \text{ lb-in.}}{2(0.125 \text{ in.})(6.3281 \text{ in.}^2)} = 2844 \text{ psi}$$

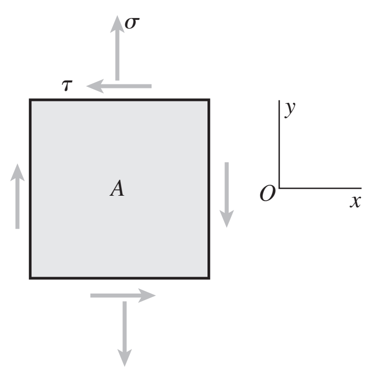
$$\sigma = \frac{Mc}{I} = \frac{(3000 \text{ lb-in.})(1.75 \text{ in.})}{2.1396 \text{ in.}^4} = 2454 \text{ psi}$$

(The shear force V produces no stresses at point A.)

STRESS ELEMENT AT POINT A

(This view is looking downward at the top of the bracket.)





$\sigma_x = 0$ $\sigma_y = \sigma = 2454$ psi
$\tau_{xy} = -\tau = -2844$ psi

PRINCIPAL STRESSES AND MAXIMUM SHEAR STRESS

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= 1227 \text{ psi} \pm 3097 \text{ psi}$$

$\sigma_1 = 4324$ psi $\sigma_2 = -1870$ psi

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3097 \text{ psi}$$

MAXIMUM TENSILE STRESS:

$\sigma_t = 4320$ psi ←

MAXIMUM COMPRESSIVE STRESS:

$\sigma_c = -1870$ psi ←

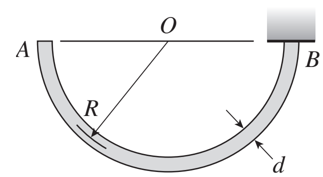
MAXIMUM SHEAR STRESS:

$\tau_{max} = 3100$ psi ←

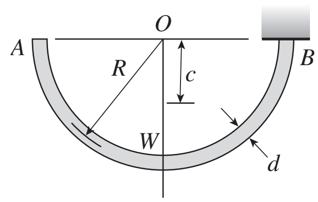
NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

Problem 8.5-12 A semicircular bar AB lying in a horizontal plane is supported at B (see figure). The bar has centerline radius R and weight q per unit of length (total weight of the bar equals πqR). The cross section of the bar is circular with diameter d.

Obtain formulas for the maximum tensile stress σ_t, maximum compressive stress σ_c, and maximum in-plane shear stress τ_{max} at the top of the bar at the support due to the weight of the bar.



Solution 8.5-12 Semicircular bar



d = diameter of bar R = radius of bar
q = weight of bar per unit length
W = weight of bar = πqR
Weight of bar acts at the center of gravity

From Case 23, Appendix D, with $\beta = \pi/2$, we get $\bar{y} = \dfrac{22}{\pi}$

$\therefore\ c = \dfrac{2R}{\pi}$

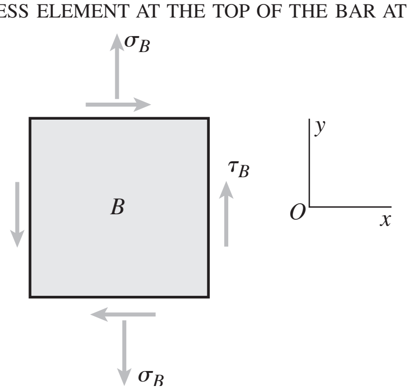
Bending moment at B: $M_B = W_C = 2qR^2$
Torque at B: $T_B = WR = \pi qR^2$
(Shear force at B produces no shear stress at the top of the bar.)

STRESSES AT THE TOP OF THE BAR AT B

$$\sigma_B = \frac{M_B(d/2)}{I} = \frac{(2qR^2)(d/2)}{\pi d^4/64} = \frac{64qR^2}{\pi d^3}$$

$$\tau_B = \frac{T_B(d/2)}{I_P} = \frac{(\pi qR^2)(d/2)}{\pi d^4/32} = \frac{16qR^2}{d^3}$$

STRESS ELEMENT AT THE TOP OF THE BAR AT B



$\sigma_x = 0$
$\sigma_y = \sigma_B$
$\tau_{xy} = \tau_B$

PRINCIPAL STRESSES:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{\sigma_B}{2} \pm \sqrt{\left(-\frac{\sigma_B}{2}\right)^2 + \tau_B^2}$$

$$= \frac{32qR^2}{\pi d^3} \pm \sqrt{\left(\frac{32qR^2}{\pi d^3}\right)^2 + \left(\frac{16qR^2}{d^3}\right)^2}$$

$$= \frac{16qR^2}{\pi d^3}(2 \pm \sqrt{4 + \pi^2})$$

MAXIMUM TENSILE STRESS

$$\sigma_t = \sigma_1 = \frac{16qR^2}{\pi d^3}(2 + \sqrt{4 + \pi^2})$$

$$= 29.15\,\frac{qR^2}{d^3} \quad \leftarrow$$

MAXIMUM COMPRESSIVE STRESS

$$\sigma_c = \sigma_2 = \frac{16qR^2}{\pi d^3}(2 + \sqrt{4 + \pi^2})$$

$$= -8.78\,\frac{qR^2}{d^3} \quad \leftarrow$$

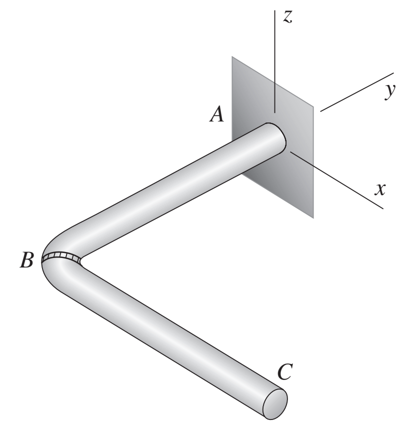
MAXIMUM IN-PLANE SHEAR STRESS (EQ. 7-26)

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{16qR^2}{\pi d^3}\sqrt{4 + \pi^2}$$

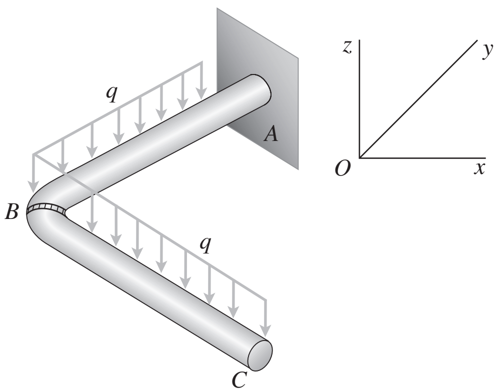
$$= 18.97\,\frac{qR^2}{d^3} \quad \leftarrow$$

Problem 8.5-13 An arm *ABC* lying in a horizontal plane and supported at *A* (see figure) is made of two identical solid steel bars *AB* and *BC* welded together at a right angle. Each bar is 20 in. long.

Knowing that the maximum tensile stress (principal stress) at the top of the bar at support *A* due solely to the weights of the bars is 932 psi, determine the diameter *d* of the bars.

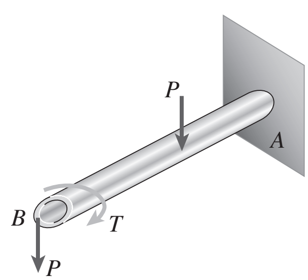


Solution 8.5-13 Horizontal arm ABC



L = length of *AB* and *BC*
d = diameter of *AB* and *BC*
A = cross-sectional area
$= \pi d^2/4$
γ = weight density of steel
q = weight per unit length of bars
$= \gamma A = \pi\gamma d^2/4$ (1)

RESULTANT FORCES ACTING ON *AB*



P = weight of *AB* and *BC*

$$P = qL = \pi\gamma L d^2/4 \quad (2)$$

T = torque due to weight of *BC*

$$T = (qL)\left(\frac{L}{2}\right) = \frac{qL^2}{2} = \frac{\pi\gamma L^2 d^2}{8} \quad (3)$$

M_A = bending moment at *A*

$$M_A = PL + PL/2 = 3PL/2 = 3\pi\gamma L^2 d^2/8 \quad (4)$$

STRESSES AT THE TOP OF THE BAR AT *A*

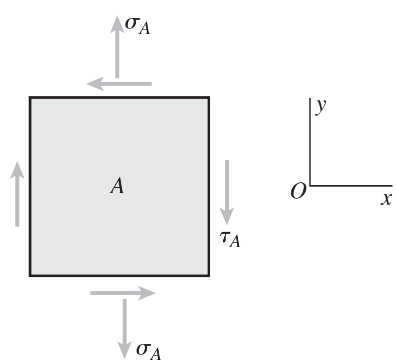
σ_A = normal stress due to M_A

$$\sigma_A = \frac{M(d/2)}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{12\gamma L^2}{d} \quad (5)$$

τ_A = shear stress due to torque T

$$\tau_A = \frac{T(d/2)}{I_p} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{2\gamma L^2}{d} \quad (6)$$

STRESS ELEMENT ON TOP OF THE BAR AT *A*



σ_1 = principal tensile stress (maximum tensile stress)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (7)$$

$$\sigma_x = 0 \quad \sigma_y = \sigma_A \quad \tau_{xy} = -\tau_A \quad (8)$$

Substitute (8) into (7):

$$\sigma_1 = \frac{\sigma_A}{2} + \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} \quad (9)$$

Substitute from (5) and (6) and simplify:

$$\sigma_1 = \frac{\gamma L^2}{d}(6 + \sqrt{40}) = \frac{2\gamma L^2}{d}(3 + \sqrt{10}) \quad (10)$$

SOLVE FOR d

$$d = \frac{2\gamma L^2}{\sigma_1}(3 + \sqrt{10}) \quad \longleftarrow \qquad (11)$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (11):

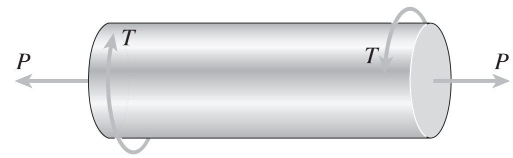
$\gamma = 490 \text{ lb/ft}^3 = 0.28356 \text{ lb/in.}^3$

$L = 20 \text{ in.} \quad \sigma_1 = 932 \text{ psi}$

$d = 1.50 \text{ in.}$ ⟵

Problem 8.5-14 A pressurized cylindrical tank with flat ends is loaded by torques T and tensile forces P (see figure). The tank has radius $r = 50$ mm and wall thickness $t = 3$ mm. The internal pressure $p = 3.5$ MPa and the torque $T = 450$ N·m.

What is the maximum permissible value of the forces P if the allowable tensile stress in the wall of the cylinder is 72 MPa?



Solution 8.5-14 **Cylindrical tank**

$r = 50 \text{ mm} \quad t = 3.0 \text{ mm} \quad p = 3.5 \text{ MPa}$

$T = 450 \text{ N} \cdot \text{m} \quad \sigma_{allow} = 72 \text{ MPa}$

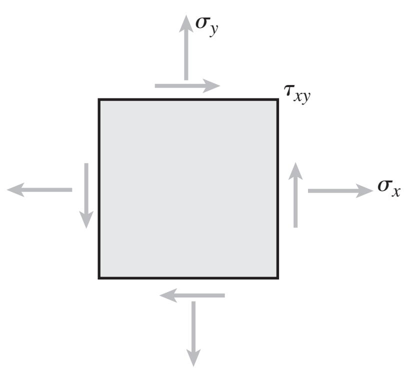
CROSS SECTION

$A = 2\pi rt = 2\pi(50 \text{ mm})(3.0 \text{ mm}) = 942.48 \text{ mm}^2$

$I_P = 2\pi r^3 t = 2\pi(50 \text{ mm})^3(3.0 \text{ mm})$

$= 2.3562 \times 10^6 \text{ mm}^4$

STRESSES IN THE WALL OF THE TANK



$$\sigma_x = \frac{pr}{2t} + \frac{P}{A}$$

$$= \frac{(3.5 \text{ MPa})(50 \text{ mm})}{2(3.0 \text{ mm})} + \frac{P}{942.48 \text{ mm}^2}$$

$= 29.167 \text{ MPa} + 1.0610 \times 10^{-3} P$

Units: σ_x = MPa, $\quad P$ = newtons

$$\sigma_y = \frac{pr}{t} = 58.333 \text{ MPa}$$

$$\tau_{xy} = -\frac{T_r}{I_P} = -\frac{(450 \text{ N} \cdot \text{m})(50 \text{ mm})}{2.3562 \times 10^6 \text{ mm}^4}$$

$= -9.5493 \text{ MPa}$

MAXIMUM TENSILE STRESS

$$\sigma_{max} = \sigma_{allow} = 72 \text{ MPa} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$72 = 43.750 + (530.52 \times 10^{-6})P$

$+ \sqrt{[-14.583 + (530.52 \times 10^{-6})P]^2 + (-9.5493)^2}$

$28.250 - 0.00053052P$

$= \sqrt{(-14.583 + 0.00053052\,P)^2 + 91.189}$

Square both sides and simplify:

$494.21 = 0.014501\,P$

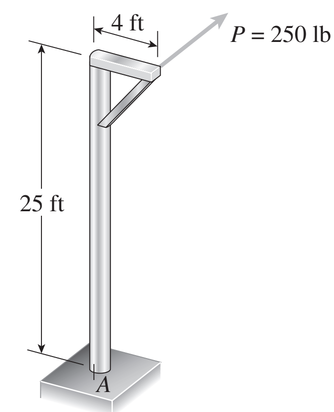
SOLVE FOR P $\quad P = 34{,}080$ N OR

$P_{max} = 34.1$ kN ⟵

Problem 8.5-15 A post having a hollow circular cross section supports a horizontal load $P = 250$ lb acting at the end of an arm that is 4 ft long (see figure on the next page). The height of the post is 25 ft, and its section modulus is $S = 10$ in.^{3}

(a) Calculate the maximum tensile stress σ_{max} and maximum in-plane shear stress τ_{max} at point A due to the load P. Point A is located on the "front" of the post, that is, at the point where the tensile stress due to bending alone is a maximum.

(b) If the maximum tensile stress and maximum in-plane shear stress at point A are limited to 16,000 psi and 6,000 psi, respectively, what is the largest permissible value of the load P?



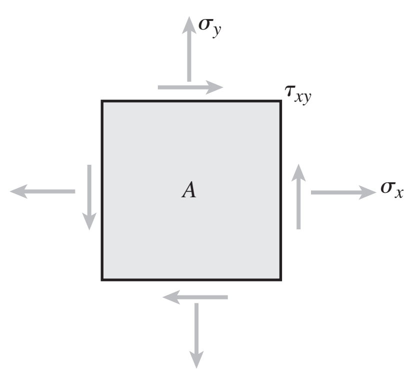
Solution 8.5-15 Post with horizontal load

$P = 250$ lb
b = length of arm
$= 4.0$ ft $= 48$ in.
h = height of post
$= 25$ ft $= 300$ in.
S = section modulus
$= 10$ in.^{3}

REACTIONS AT THE SUPPORT

$M = Ph = 75{,}000$ lb-in.
$T = Pb = 12{,}000$ lb-in.
$V = P = 250$ lb

STRESSES AT POINT A



$\sigma_x = 0$

$$\sigma_y = \frac{M}{S} = \frac{75{,}000 \text{ lb-in.}}{10 \text{ in.}^3} = 7500 \text{ psi}$$

$$\tau_{xy} = \frac{Tr}{I_P}$$

r = outer radius of post

$$S = \frac{I}{r} = \frac{I_P}{2r} \quad \therefore \ \tau_{xy} = \frac{T}{2S} = \frac{12{,}000 \text{ lb-in.}}{2(10 \text{ in.}^3)} = 600 \text{ psi}$$

(The shear force V produces no stresses at point A.)

(a) MAXIMUM TENSILE STRESS AND MAXIMUM SHEAR STRESS

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 3750 \text{ psi} + \sqrt{(3750 \text{ psi})^2 + (600 \text{ psi})^2}$$

$$= 3750 \text{ psi} + 3798 \text{ psi} = 7550 \text{ psi} \leftarrow$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3800 \text{ psi} \leftarrow$$

(b) ALLOWABLE LOAD P

$\sigma_{allow} = 16{,}000$ psi $\quad \tau_{allow} = 6{,}000$ psi

The stresses at point A are proportional to the load P.

Based on tensile stress:

$$\frac{P_{allow}}{P} = \frac{\sigma_{allow}}{\sigma_{max}} \quad P_{allow} = (250 \text{ lb})\left(\frac{16{,}000 \text{ psi}}{7{,}550 \text{ psi}}\right) = 530 \text{ lb}$$

Based on shear stress:

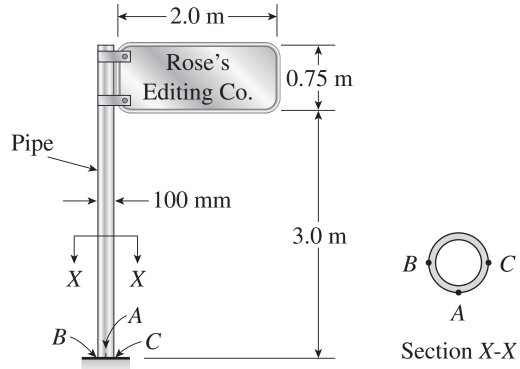
$$\frac{P_{allow}}{P} = \frac{\tau_{allow}}{\tau_{max}} \quad P_{allow} = (250 \text{ lb})\left(\frac{6{,}000 \text{ psi}}{3{,}800 \text{ psi}}\right) = 395 \text{ lb}$$

Shear stress governs:

$P_{allow} = 395$ lb $\leftarrow$

Problem 8.5-16 A sign is supported by a pipe (see figure) having outer diameter 100 mm and inner diameter 80 mm. The dimensions of the sign are 2.0 m × 0.75 m, and its lower edge is 3.0 m above the base. Note that the center of gravity of the sign is 1.05 m from the axis of the pipe. The wind pressure against the sign is 1.5 kPa.

Determine the maximum in-plane shear stresses due to the wind pressure on the sign at points A, B, and C, located on the outer surface at the base of the pipe.



Solution 8.5-16 **Sign supported by a pipe**

PIPE: $d_2 = 100$ mm $\quad d_1 = 80$ mm $\quad t = 10$ mm

SIGN: 2.0 m × 0.75 m $\quad A = 1.50\ \text{m}^2$

h = height from the base to the center of gravity of the sign

$$h = 3.0\ \text{m} + \frac{1}{2}(0.75\ \text{m}) = 3.375\ \text{m}$$

b = horizontal distance from the center of gravity of the sign to the axis of the pipe

$$b = \frac{1}{2}(2.0\ \text{m}) + \frac{1}{2}(100\ \text{mm}) = 1.05\ \text{m}$$

WIND PRESSURE: $p = 1.5$ kPa

P = horizontal wind force on the sign

$= pA = (1.5\ \text{kPa})(1.50\ \text{m}^2)$

$= 2250$ N

STRESS RESULTANTS AT THE BASE

$M = Ph = (2250\ \text{N})(3.375\ \text{m}) = 7593.8\ \text{N}\cdot\text{m}$

$T = Pb = (2250\ \text{N})(1.05\ \text{m}) = 2362.5\ \text{N}\cdot\text{m}$

$V = P = 2250$ N

PROPERTIES OF THE TUBULAR CROSS SECTION

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 2.8981 \times 10^6\ \text{mm}^4$$

$$I_P = 2I = 5.7962 \times 10^6\ \text{mm}^4$$

$$Q = \frac{2}{3}(r_2^3 - r_1^3) = \frac{1}{12}(d_2^3 - d_1^3) = 40.667 \times 10^3\ \text{mm}^3$$

(From Eq. 5-43b, Chapter 5)

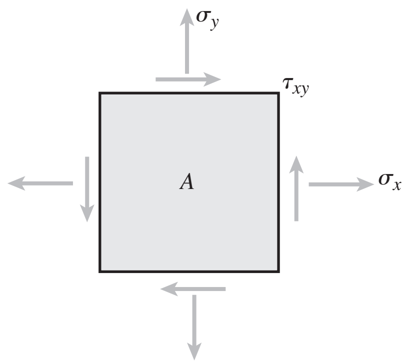
STRESSES AT POINT A

$$\sigma_x = 0$$

$$\sigma_y = \frac{Mc}{I} = \frac{Md_2}{2I}$$

$$= \frac{(7593.8\ \text{N}\cdot\text{m})(0.1\ \text{m})}{2(2.8981 \times 10^6\ \text{mm}^4)}$$

$$= 131.01\ \text{MPa}$$



$$\tau_{xy} = \frac{Tr}{I_P} = \frac{Td_2}{2I_P} = \frac{(2362.5\ \text{N}\cdot\text{m})(0.1\ \text{m})}{2(5.7962 \times 10^6\ \text{mm}^4)}$$

$$= 20.380\ \text{MPa}$$

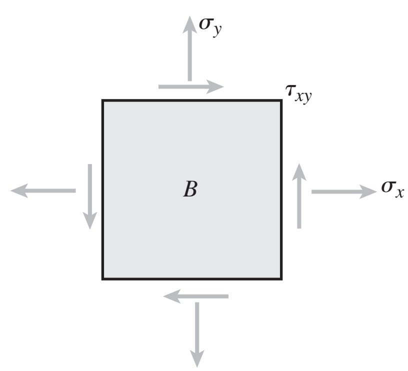
$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(65.507\ \text{MPa})^2 + (20.380\ \text{MPa})^2}$$

$$= 68.60\ \text{MPa}$$

$$\tau_A = 68.6\ \text{MPa} \quad \longleftarrow$$

STRESSES AT POINT B



$\sigma_y = 0$ (Moment M produces no stresses at points B and C)

$\sigma_x = 0$

$$\tau_{xy} = \frac{Tr}{I_P} - \frac{VQ}{Ib}$$

$$\frac{Tr}{I_P} = 20.380 \text{ MPa}$$

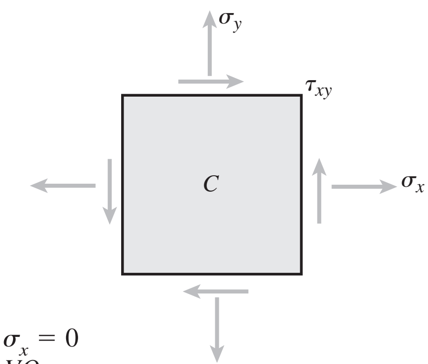
$$\frac{VQ}{Ib} = \frac{(2250 \text{ N})(40.667 \times 10^3 \text{ mm}^3)}{(2.8981 \times 10^6 \text{ mm}^4)(20 \text{ mm})}$$

$$= 1.5786 \text{ MPa}$$

$\tau_{xy} = 20.380 \text{ MPa} - 1.5786 \text{ MPa} = 18.80 \text{ MPa}$

Pure shear. $\tau_B = 18.8$ MPa ←

STRESSES AT POINT C



$\sigma_y = 0 \qquad \sigma_x = 0$

$$\tau_{xy} = \frac{Tr}{I_P} + \frac{VQ}{Ib}$$

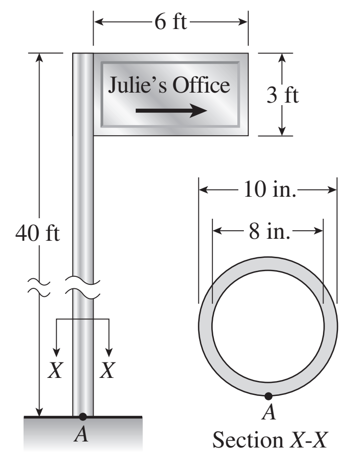
$\tau_{xy} = 20.380 \text{ MPa} + 1.5786 \text{ MPa} = 21.96 \text{ MPa}$

Pure shear. $\tau_C = 22.0$ MPa ←

Problem 8.5-17 A sign is supported by a pole of hollow circular cross section, as shown in the figure. The outer and inner diameters of the pole are 10.0 in. and 8.0 in., respectively. The pole is 40 ft high and weighs 3.8 k. The sign has dimensions 6 ft × 3 ft and weighs 400 lb. Note that its center of gravity is 41 in. from the axis of the pole. The wind pressure against the sign is 30 lb/ft^2.

(a) Determine the stresses acting on a stress element at point A, which is on the outer surface of the pole at the "front" of the pole, that is, the part of the pole nearest to the viewer.

(b) Determine the maximum tensile, compressive, and shear stresses at point A.



8.5-17 Sign supported by a pole

POLE: $d_2 = 10$ in. $\quad d_1 = 8$ in.

W_1 = weight of pole
$= 3800$ lb

SIGN: 6 ft × 3 ft, or 72 in. × 36 in.

$A = 18 \text{ ft}^2 = 2592 \text{ in.}^2$

W_2 = weight of sign = 400 lb

h = height from the base to the center of gravity of the sign

$h = 40 \text{ ft} - 1.5 \text{ ft} = 38.5 \text{ ft} = 462$ in.

b = horizontal distance from the center of gravity of the sign to the axis of the pole

$$b = \frac{1}{2}(6 \text{ ft}) + \frac{1}{2}(d_2) = 41 \text{ in.}$$

WIND PRESSURE: $p = 30 \text{ lb/ft}^2 = 0.208333$ psi

P = horizontal wind force on the sign

$= pA = (0.208333 \text{ psi})(2592 \text{ in.}^2)$

$= 540$ lb

STRESS RESULTANTS AT THE BASE

Axial force: $N = w_1 + w_2 = 4200$ lb (compression)

Bending moment from wind pressure:

$M = Ph = (540 \text{ lb})(462 \text{ in.}) = 249{,}480$ lb-in.

(This moment causes tension at point A.)

Bending moment from weight of sign:

(This moment causes zero stress at point A.)

Torque from wind pressure:

$T = Pb = (540 \text{ lb})(41 \text{ in.}) = 22{,}140$ lb-in.

Shear force from wind pressure:

(This force causes zero shear stress at point A.)

PROPERTIES OF THE TUBULAR CROSS SECTION

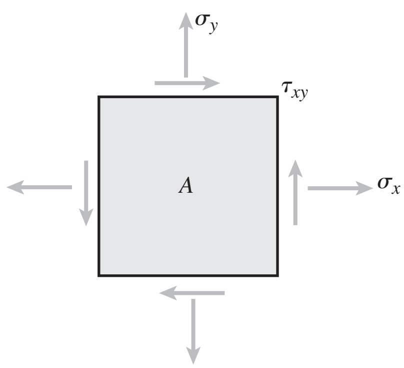
$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 28.274 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 289.81 \text{ in.}^4$$

$$I_p = 2I = 579.62 \text{ in.}^4$$

$$c = \frac{d_2}{2} = 5.0 \text{ in.}$$

(a) STRESSES AT POINT A



$$\sigma_x = 0 \quad \longleftarrow$$

$$\sigma_y = -\frac{N}{A} + \frac{Mc}{I}$$

$$\tau_{xy} = \frac{Td_2}{2\,Ip}$$

$$\sigma_y = -\frac{4200 \text{ lb}}{28.274 \text{ in.}^2} + \frac{(249{,}480 \text{ lb-in.})(5.0 \text{ in.})}{289.81 \text{ in.}^4}$$

$$= -148.5 \text{ psi} + 4{,}304.2 \text{ psi} = 4156 \text{ psi} \quad \longleftarrow$$

$$\tau_{xy} = \frac{(22{,}140 \text{ lb-in.})(10 \text{ in.})}{2(579.62 \text{ in.}^4)} = 191 \text{ psi} \quad \longleftarrow$$

(b) MAXIMUM STRESSES AT POINT A

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 2078 \text{ psi} \pm 2087 \text{ psi}$$

$$\sigma_1 = 4165 \text{ psi} \qquad \sigma_2 = -9 \text{ psi}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2087 \text{ psi}$$

Max. tensile stress: $\sigma_t = 4165$ psi $\longleftarrow$

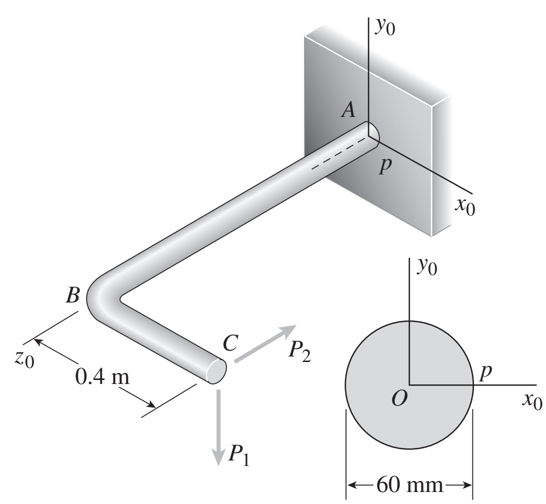
Max. compressive stress: $\sigma_c = -9$ psi $\longleftarrow$

Max. shear stress: $\tau_{max} = 2087$ psi $\longleftarrow$

NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

Problem 8.5-18 A horizontal bracket *ABC* (see figure on the next page) consists of two perpendicular arms *AB* and *BC*, the latter having a length of 0.4 m. Arm *AB* has a solid circular cross section with diameter equal to 60 mm. At point *C* a load $P_1 = 2.02$ kN acts vertically and a load $P_2 = 3.07$ kN acts horizontally and parallel to arm *AB*.

Considering only the forces P_1 and P_2, calculate the maximum tensile stress σ_t, the maximum compressive stress σ_c, and the maximum in-plane shear stress τ_{max} at point *p*, which is located at support *A* on the side of the bracket at midheight.



Cross section at *A*

Solution 8.5-18 Horizontal bracket

P_1 = vertical force
= 2.02 kN
P_2 = horizontal force
= 3.07 kN
b = length of arm BC
= 0.4 m
d = diameter of solid bar
= 60 mm

PROPERTIES OF THE CROSS SECTION

$$A = \frac{\pi d^2}{4} = 2827.4 \text{ mm}^2$$

$$I = \frac{\pi d^4}{64} = 636{,}170 \text{ mm}^4$$

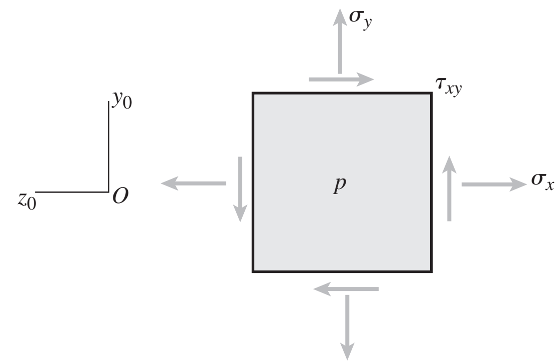
$$I_p = 2I = 1272.3 \times 10^3 \text{ mm}^4$$

$$c = \frac{d}{2} = 30\,\text{mm} \qquad r = \frac{d}{2} = 30\,\text{mm}$$

STRESS RESULTANTS AT SUPPORT A

$N = P_2 = 3070$ N (compression)
$M_y = P_2 b = 1228$ N · m
M_x may be omitted because it produces no stresses at point p.
$T = P_1 b = 808$ N · m
$V = P_1 = 2020$ N

STRESSES AT POINT p ON THE SIDE OF THE BRACKET



$$\sigma_x = -\frac{N}{A} - \frac{M_y c}{I}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{Tr}{I_p} + \frac{4V}{3A}$$

$$\sigma_x = -\frac{3070\,\text{N}}{2827.4\,\text{mm}^2} - \frac{(1228\,\text{N}\cdot\text{m})(30\,\text{mm})}{636{,}170\,\text{mm}^4}$$

$$= -1.0858 \text{ MPa} - 57.909 \text{ MPa}$$

$$= -58.995 \text{ MPa}$$

$$\tau_{xy} = \frac{(808\,\text{N}\cdot\text{m})(30\,\text{mm})}{1272.3 \times 10^3\,\text{mm}^4} + \frac{4(2020\,\text{N})}{3(2827.4\,\text{mm}^2)}$$

$$= 19.051 \text{ MPa} + 0.953 \text{ MPa} = 20.004 \text{ MPa}$$

MAXIMUM STRESSES AT POINT P

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -29.498 \text{ MPa} \pm 35.641 \text{ MPa}$$

$$\sigma_1 = 6.1 \text{ MPa} \qquad \sigma_2 = -65.1 \text{ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.6 \text{ MPa}$$

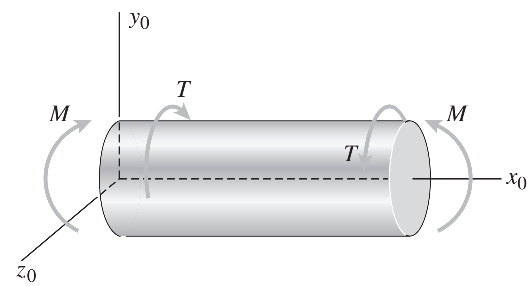
Max. tensile stress: $\sigma_t = 6.1$ MPa ←

Max. compressive stress: $\sigma_c = -65.1$ MPa ←

Max. in-plane shear stress: $\tau_{max} = 35.6$ MPa ←

Problem 8.5-19 A cylindrical pressure vessel with flat ends is subjected to a torque T and a bending moment M (see figure). The outer radius is 12.0 in. and the wall thickness is 1.0 in. The loads are as follows: $T = 800$ k-in., $M = 1000$ k-in., and the internal pressure $p = 900$ psi.

Determine the maximum tensile stress σ_t, maximum compressive stress σ_c, and maximum shear stress τ_{max} in the wall of the cylinder.



Solution 8.5-19 Cylindrical pressure vessal

Internal pressure: $p = 900$ psi
Bending moment: $M = 1000$ k-in.
Torque: $T = 800$ k-in.
Outer radius: $r_2 = 12$ in.
Wall thickness: $t = 1.0$ in.
Mean radius: $r = r_2 - t/2 = 11.5$ in.
Outer diameter: $d_2 = 24$ in.
Inner diameter: $d_1 = 22$ in.

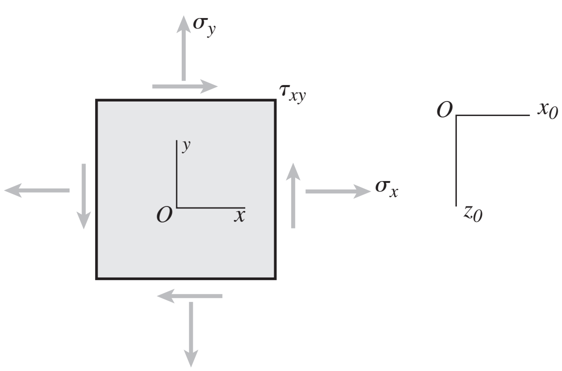
MOMENT OF INERTIA

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 4787.0 \text{ in.}^4$$

$$I_p = 2I = 9574.0 \text{ in.}^4$$

NOTE: Since the stresses due to T and p are the same everywhere in the cylinder, the maximum stresses occur at the top and bottom of the cylinder where the bending stresses are the largest.

PART (a). TOP OF THE CYLINDER



Stress element on the top of the cylinder as seen from above.

$$\sigma_x = \frac{pr}{2t} - \frac{Mr_3}{I} = 5175.0 \text{ psi} - 2506.8 \text{ psi}$$

$$= 2668.2 \text{ psi}$$

$$\sigma_y = \frac{pr}{t} = 10{,}350 \text{ psi}$$

$$\tau_{xy} = -\frac{Tr_2}{I_p} = -1002.7 \text{ psi}$$

PRINCIPAL STRESSES

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 6509.1 \text{ psi} \pm 3969.6 \text{ psi}$$

$\sigma_1 = 10{,}479$ psi $\quad \sigma_2 = 2540$ psi

MAXIMUM SHEAR STRESSES

In-plane: $\tau = 3970$ psi

Out-of-plane:

$$\tau = \frac{\sigma_1}{2} \quad \text{or} \quad \frac{\sigma_2}{2} \quad \tau = \frac{\sigma_1}{2} = 5240 \text{ psi}$$

$\therefore \tau_{max} = 5240$ psi

MAXIMUM STRESSES FOR THE TOP OF THE CYLINDER

$\sigma_t = 10{,}480$ psi $\quad \sigma_c = 0$ (No compressive stresses)

$\tau_{max} = 5240$ psi

PART (b). BOTTOM OF THE CYLINDER

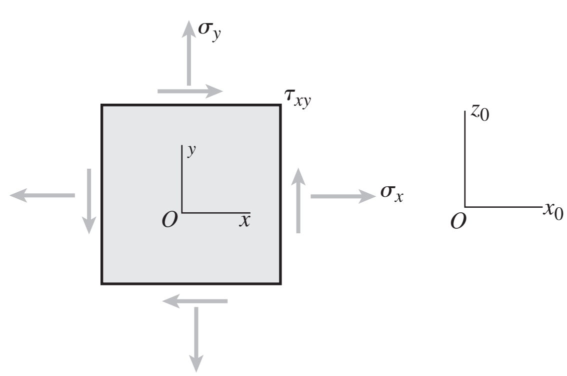
Stress element on the bottom of the cylinder as seen from below.

$$\sigma_x = \frac{pr}{2t} + \frac{Mr_2}{I} = 5175.0 \text{ psi} + 2506.8 \text{ psi}$$

$$= 7681.8 \text{ psi}$$

$$\sigma_y = \frac{pr}{t} = 10{,}350 \text{ psi}$$

$$\tau_{xy} = -\frac{Tr_2}{I_p} = -1002.7 \text{ psi}$$



PRINCIPAL STRESSES

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$= 9015.9 \text{ psi} \pm 1668.9 \text{ psi}$

$\sigma_1 = 10{,}685 \text{ psi} \quad \sigma_2 = 7347 \text{ psi}$

MAXIMUM SHEAR STRESSES

In-plane: $\tau = 1669$ psi

Out-of-plane:

$$\tau = \frac{\sigma_1}{2} \text{ or } \frac{\sigma_2}{2} \quad \tau = \frac{\sigma_1}{2} = 5340 \text{ psi}$$

$\therefore \tau_{max} = 5340$ psi

MAXIMUM STRESSES FOR THE BOTTOM OF THE CYLINDER

$\sigma_t = 10{,}680$ psi $\quad \sigma_c = 0$ (No compressive stresses)

$\tau_{max} = 5340$ psi

PART (c). ENTIRE CYLINDER

The largest stresses are at the bottom of the cylinder.

$\sigma_t = 10{,}680$ psi ←

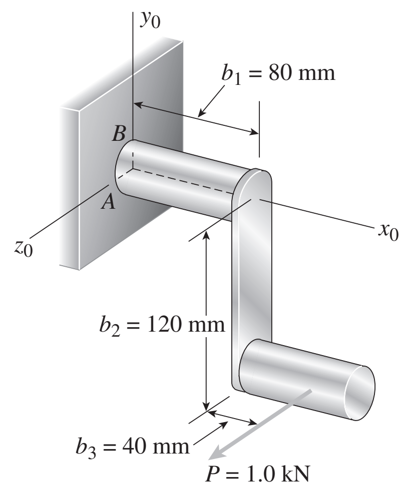
$\sigma_c = 0$ (No compressive stresses) ←

$\tau_{max} = 5340$ psi ←

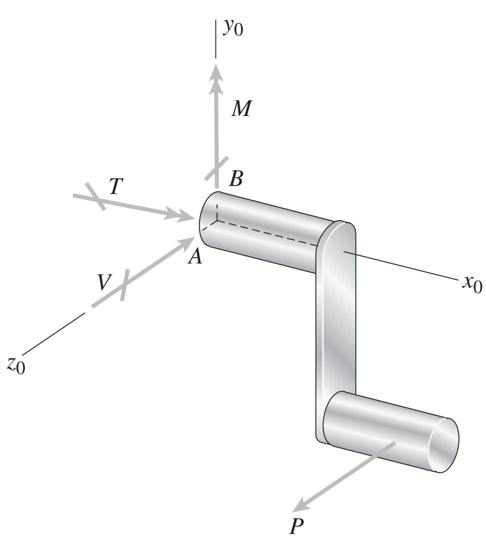
Problem 8.5-20 For purposes of analysis, a segment of the crankshaft in a vehicle is represented as shown in the figure. The load P equals 1.0 kN, and the dimensions are $b_1 = 80$ mm, $b_2 = 120$ mm, and $b_3 = 40$ mm. The diameter of the upper shaft is $d = 20$ mm.

(a) Determine the maximum tensile, compressive, and shear stresses at point A, which is located on the surface of the upper shaft at the z_0 axis.

(b) Determine the maximum tensile, compressive, and shear stresses at point B, which is located on the surface of the shaft at the y_0 axis.



Solution 8.5-20 Part of a crankshaft



DATA $\quad P = 1.0$ kN $\quad d = 20$ mm

$b_1 = 80$ mm $\quad b_2 = 120$ mm

$b_3 = 40$ mm

REACTIONS AT THE SUPPORT

M = moment about the y_0 axis

(M produces compression at point A and no stress at point B)

$M = P(b_1 + b_3) = 120 \text{ N} \cdot \text{m}$

T = torque about the x_0 axis

(T produces shear stresses at points A and B)

$T = Pb_2 = 120 \text{ N} \cdot \text{m}$

V = force directed along the z_0 axis

(V produces shear stress at point B and no stress at point A)

$V = P = 1000$ N

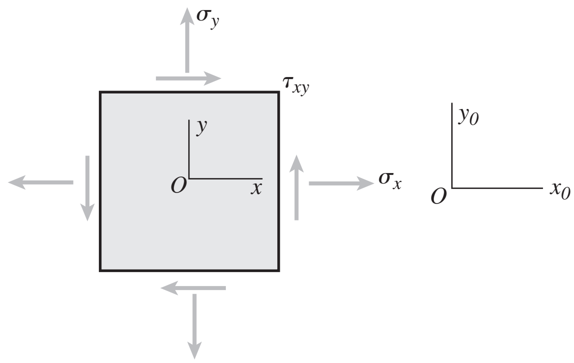
MOMENTS OF INERTIA AND CROSS-SECTIONAL AREA

$$I = \frac{\pi d^4}{64} = 7{,}854.0 \text{ mm}^4$$

$$I_p = 2I = 15{,}708.0 \text{ mm}^4$$

$$A = \frac{\pi d^2}{4} = 314.16 \text{ mm}^2$$

(a) STRESSES AT POINT *A*



$$\sigma_x = -\frac{Md}{2I} = -152.79 \text{ MPa}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{Td}{2I_p} = 76.394 \text{ MPa}$$

MAXIMUM STRESSES AT POINT *A*

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -76.40 \text{ MPa} \pm 108.04 \text{ MPa}$$

$\sigma_1 = 31.64$ MPa $\quad \sigma_2 = -184.44$ MPa

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 108.04 \text{ MPa}$$

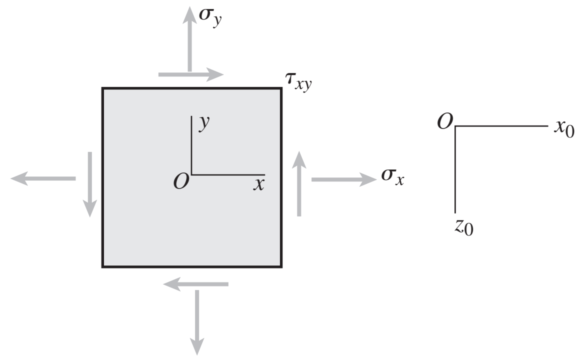
Max. tensile stress: $\sigma_t = 32$ MPa ⟵

Max. compressive stress: $\sigma_c = -184$ MPa ⟵

Max. shear stress: $\tau_{max} = 108$ MPa ⟵

NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

(b) STRESSES AT POINT *B*



$\sigma_x = 0 \quad \sigma_y = 0$

$$\tau_{xy} = \frac{T_d}{2I_p} - \frac{4V}{3A} = 76.394 \text{ MPa} - 4.244 \text{ MPa}$$

$$= 72.15 \text{ MPa}$$

MAXIMUM STRESSES AT POINT *B*

Element is in PURE SHEAR.

$\sigma_1 = 72.2$ MPa $\quad \sigma_2 = -72.2$ MPa

$\tau_{max} = 72.2$ MPa

Max. tensile stress: $\sigma_t = 72.2$ MPa ⟵

Max. compressive stress: $\sigma_c = -72.2$ MPa ⟵

Max. shear stress: $\tau_{max} = 72.2$ MPa ⟵

NOTE: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.